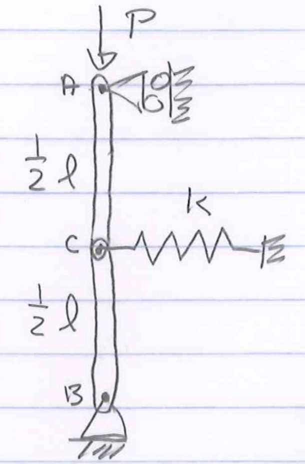
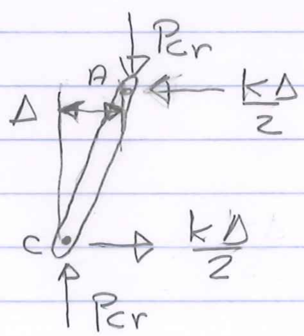
34. Basic idea of stability of a column



The column consists of two rigid links pinned together with pinned supports. The top support can move down freely. A horizontal spring is attached to the column at mid-height. (The spring force is assumed to remain horizontal as the column buckles down.)

We want to find the buckling value of P, denoted by P_{cr}. The load $P = P_{cr}$ is able to hold the column in a slightly deformed position with small lateral deflection Δ at mid-height. Examine a FBD of the upper link in a deformed position with $P = P_{cr}$.



$$\Sigma M_c = 0 = P_{cr} \cdot \Delta - \frac{k\Delta}{2} \cdot \frac{l}{2} = 0$$

$$P_{cr} = \frac{kl}{4} \quad (\text{small } \Delta \text{ assumed})$$

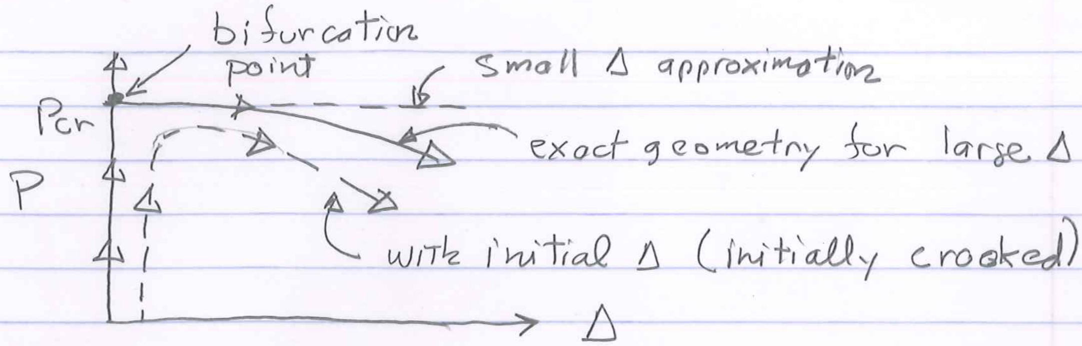
A plot of P vs. Δ can be obtained as:

$\Delta = 0$ for $P < P_{cr}$

for $\Delta > 0$, use the FBD above with P replacing P_{cr} and consider the exact geometry as Δ becomes large.

$$\Sigma M_c = 0 = P \cdot \Delta - \frac{k\Delta}{2} \cdot \left[\frac{l^2}{4} - \Delta^2\right]^{\frac{1}{2}}$$

so,
$$P = \frac{kl}{4}\left[1 - \frac{4\Delta^2}{l^2}\right]^{\frac{1}{2}}$$



The above plot also included the case where the column is initially crooked. As shown, there is no sudden bifurcation as for the straight column, and the buckling load P_{cr} is never reached.